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Estimates of Birth Rates for Some African Countries with Two Censuses

Introduction

BOTH data collection systems and techniques of estimation in the study of fertility in sub-Saharan African countries have improved since the 1960's. In many sub-Saharan African countries there have already been at least two censuses. The 1980 round of censuses and future censuses will lead to changes in the data available for fertility in this region. In this paper two recent methods to estimate birth rate based on two census age-sex distributions are examined with reference to data situation in sub-Saharan Africa. The object of this paper is to estimate birth rate by the two methods, to be described below, for some English speaking African countries with two censuses, and compare their relative strengths in the statistically less developed African Countries.

Methods of Estimating Birth Rates

For a closed population experiencing constant fertility and mortality an estimate of birth rate is easily obtained by using model stable populations. Many procedures are available in the literature for this purpose, which involve matching some selected characteristics of the observed population with those of the stable model. One such method suggested by Coale (1981) involving the use of the proportion of the population of both sexes under age 15, C(15), and l_5 as the matching criteria appears to be promising. When a population experiences declines in fertility and/or mortality the stability of the population will be

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disturbed and the usual stable birth rate estimates will have to oe adjusted for these changes, Coale (1981) suggested an adjustment which is found to be robust and at the same time computationally simple. In recent years Preston and Coale (1982) and Preston (1983) have generalized the stable population equations that can be applied to all populations—stable or non-stable. It is also possible to adjust the stable birth rate estimates using the framework of generalized stable population equations. These two procedures of obtaining birth rates assume importance for situations where stability conditions, namely, constant age specific fertility rates and mortality rates with no migration are not valid. Before proceeding to illustrate the applications of the two methods, a brief review of the two methods and their interrelationship will be discussed.

Coale Method

Coale (1981) suggested the use of the observed C(I5) for both sexes and l_5 to locate an appropriate stable model from a family of stable models to represent the observed population and use its birth rate as an estimate of the study population. The estimate of l_5 can be obtained by any of the indirect methods like the Brass method. It was observed by Coale that this method of estimation yields birth rates that are not very much affected even when the populations are not stable. Further, he also suggested an adjustment for the stable birth rate for non-stability which is given below :

$$b_c = b_s \cdot \text{Exp}\,[7.5\,(r_0 - r_s)], \qquad (1)$$

where b_c is the adjusted birth rate by Coale method; b_s, is the stable birth rate corresponding to C(15) and l_5; and r_s, and r_0 are the rates of increase in the stable and study populations respectively.

The logic behind the above adjustment is explained by Coale by treating the estimation of stable birth rate from C(15) and l_5 as a form of reverse survival method that gives an estimate of the average birth rate during the 15 years preceding the census. The persons under age 15 when reverse survived by life table survival ratios corresponding to estimated l_5 lead to the births during the *15* years preceding the census. To obtain the birth rate we need the denominator, namely, the person years Jived during the preceding 15 years. These person years lived are obtained by using the rate of increase r which differs for a stable and non-stable or observed population. For example, the person years lived in the stable population are obtained as $P_0 \cdot \text{Exp}\,[-7.5\, r_s]$ and for the non-stable observed population as $P_0 \cdot \text{Exp}\,[-7.5\, r_0]$. The value 7.5 is the number of years before the census where the mid-point occurs and P_0 it the total current population. Thus the person years ia the stable tituation and hence the stable birth rate can be adjusted by the factor Exp [7.5 [r, — r,)] to take care of the non-stable situation. For more details the reader should refer to Coale (1981) or United Nations (1983). Since *b,* is the average value for fifteen years prior to the most recent census data, T_t and l_5 refers to the time about 6-7 years prior to T, b_e should be treated to refer to the time $(T - 7.5)$.

Preston- Coale Method Based on Generalized Stable Equations

It is possible to derive the adjustment factor for the stable birth rate for non-stability using Preston-Coale generalized stable equations. For a non-stable population we have ;

$$c(a) = b_n \cdot \text{Exp}\left[-\int_0^a r(x)\,dx\right] \cdot p(a), \tag{2}$$

where b_n is the birth rate of the observed non-stable population, $r(x)$ are the age specific rate of increase of the population estimated from two censuses and $p(a)$ the probability of survival from birth to exact age a (Preston and Coale 1982). It follows from equation (2) that the cumulated population under age 15isgivenby

$$C(15) = \int_0^{15} c(a)\,da = b_n \int_0^{15} \text{Exp}\left[-\int_0^a r(x)\,dx\right] \cdot p(a)\,da. \tag{3}$$

From the above equation we have

$$b_n = C(15) \Big/ \int_0^{15} \text{Exp}\left[-\int_0^a r(x)\,dx\right] \cdot p(a)\,da. \tag{4}$$

The equation that corresponds to equation (4) in the case of a stable population is given by

$$b_s^* = C(15) \Big/ \int_0^{15} \text{Exp}\left[-a \cdot r_s^*\right] \cdot p(a)\,da. \tag{5}$$

It is to be noted that the $p(a)$ in both the equations (4) and (5) are obtained from a life table that corresponds to the current estimate of l_5. From equations (4) and (5) we have

$$b_n = b_s^* \int_0^{15} \text{Exp}\left[-r_s^* \cdot a\right] \cdot p(a)\,da \Big/ \int_0^{15} \text{Exp}\left[-\int_0^a r(x)\,dx\right] \cdot p(a)\,da. \tag{6}$$

As a first approximation we can write this equation as

$$b_n = b_s^* \cdot \text{Exp}\left[-(7.5\, r_s^*)\right] \int p(a)\,da / \text{Exp}\left[-\int r(x)\,dx\right] \cdot \int p(a)\,da. \tag{7}$$

This can be simplified to

$$b_n = b_s^* \cdot \text{Exp}\left[7.5\,(r_w - r_s^*)\right], \tag{8}$$

where $r_w = 1/7.5\,[5r_1 + 2.5r_2]$; r_1, r_2 are rates of increase for the age groups

0-4 and 5-9 respectively; and r_w is the weighted mean of these two rates of increase. r_w can be treated as the rate of increase for the age group 0-7.5 years.

A better approximation to equation (6), theoretically speaking, can be obtained as follows : $b_n^{*} = b_s^{*} \cdot F$, where F the adjustment factor can be obtained as

$$\frac{{}_5L_0 \cdot \mathrm{Exp}\,[-2.5r_s^*] + {}_5L_5 \cdot \mathrm{Exp}\,[-7.5r_s^*] + {}_5L_{10} \cdot \mathrm{Exp}\,[-12.5r_s^*]}{{}_5L_0 \cdot \mathrm{Exp}\,[a_1] + {}_5L_5 \cdot \mathrm{Exp}\,[a_2] + {}_5L_{10} \cdot \mathrm{Exp}\,[a_3]} \qquad (9)$$

The values of a_1, a_2.and a_3 are : $a_1 = 2.5r_1$; $a_2 = 5r_1 + 2.5r_2$,; and $a_3 = 5r_1 + 5r_2 + 2.5r_3$; where r_3 stands for observed intercensal rate of increase of the age group 10-14.

Equations (1) and (8) are similar in structure. If the values of C(15) for the two recent censuses have not substantially changed, though they might have undergone destabilization earlier on, we have $b^*_s = b_s$ and $r_s^* = r_s$ leading to

$$b'_n = b_s \cdot \mathrm{Exp}\,[7.5\,(r_w - r_s)] \qquad (10)$$

A comparison of this equation with that of equation (1) indicates that they are very similar, and in most cases r_w is close to r_0 in which case the two equations arc identical. In the trivial case of the stable situation when $r_0 = r_s = r_w = r_s^*$ both the birth rates b_e and b_n will be equal.

However, there is one difference, namely, the time reference for b_e and b_n.

As mentioned earlier, b_t obtained by Coale adjustment, refers to the time of about (T — 7.5) years. In the case of Preston-Coale framework it is necessary for C(15), l_5 and $r(x)$ to be current estimate, all referring to the same time period. Consequently, the resultant b_n will refer to time T. Thus a comparison of b_t and b^* should be made after the differences in their time reference is taken care of.

There are a number of ways to do this. One of the simplest things to do is to project back the sex-age distributions of the most recent census, using intercensal age-specific rates of growth, to the time $(T—7.5)$. Regarding r(x), one can assume a constant rate of increase between the censuses in which case it is applicable to time (T—7.5). However, if one has data for more than one census $r(x_t\ t)$ function can be fitted and $r(x,\ T—7.5)$ estimated. If single year data are available and reliable the method suggested by Coale can be used (Coale, 1984). This method has the additional advantage of smoothing the irregularities in r(x,t) where errors occur at specific ages in one census. From the results of this paper it appears that the assumption of a constant intercensal $r(x)$ is quite satisfactory. [In view of substantial differentials in the coverage of successive censuses in the developing countries estimation of a trend in $r(x)$ is subject to errors). The derived b_a with the use of $C'(15)$, $l5$ and $r(x)$ will now refer to (T—7.5) and is comparable to b_e.

Application of the Method to Data from Some African Countries

For a comparative study of the estimates of adjusted birth rates by various methods outlined above, eight countries in sub-Saharan Africa having data for two censuses have been selected. In the left half of Table 1 values of C(15) for the two censuses, as well as C'(15) obtained by reverse projecting the most recent census age-sex distributions to time (T—7.5) are shown. The intercensal period is also indicated in the table. The life table values of l5 are those obtained by using Brass method of estimating mortality indirectly from the data on the children ever born and surviving obtained at the second Census for the countries under study.

The procedure of using equation (1) is straight forward. The intercensal rate of growth r,, and the values of *r*, and *b*, corresponding to the observed C(D) of the latest census (both sexes) and l_5 are obtained, leading to *bc* by substitution in equation (1). To apply equations (8) to (10) we need the age specific rates of growth between the two censuses, namely, r_1, r_2 and r_3 respectively for 0-4, 5-9 and 10-14 age groups. Using the C'(15) obtained by reverse projecting the latest census and the l5, rs* and *bs** are obtained from the Model Stable Populations. In this paper Coale-Demeny North Family Stable Populations are used. For using equation (9) Coale-Demeny North Family Life Tables are used.

Table 2 presents the result of applying equation (I) for nine countries using the respective C(15) of the latest census, and l5. The quantities used in the computation are shown in columns (2) to (4). Columns (6) to (8) indicate the values used for each country using *C'(15)* and other values in equation (8).

Results obtained by the application of equations (9) and (10) are shown in columns (10) and (11) respectively. In column (12) the result of applying Preston integrated method to the female population of the latest census are shown. The Preston integrated method used here is the same as that given by Preston (198?) except that the fitting procedure is slightly different (Tesfay Teklu, 1985).

Discussion

A comparison of figures in columns (5) and (9) of Table 2 indicates that except for Ghana* Sierra Leone and Liberia the adjusted birth rates *be* and *b«* are very close. Interestingly, the countries that show larger deviations come from West Africa whereas countries from East and Southern Africa give close results. This could be due partly to differential census reporting in this region and higher level of mortality compared to the other two regions. The closeness is very striking in view of the fact that the age-sex data are not adjusted in any way. The use of equation (9) brought (he two even closer in most cases. By and large the use of equation (9) with C(l5) of the latest census instead of C'(15) also gave resonably close estimates to those obtained by Coale's adjust -

TABLE 1—INDICES OF MORTALITY, AGE DISTRIBUTION AND TIME PERIODS OF THE CENSUSES

Country	*Observed Populations*						*United Nations Projections 2015-2025*			
	Period	l_5	*First Census C(15)*	*Second Census C(15)*	*7.5 Years before T, C′(15)*	r_0	*C(15) for 2015*	*C(15) for 2025*	*C′(15) for 2017.5*	r_0 *for 2015-25*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ghana	60-70	.792	47.1	48.2	47.4	2.88	38.1	30.3	36.1	1.66
Liberia	62-74	.777	37.8	41.8	39.2	3.07	42.1	35.3	40.4	2.09
Sierra Leone	63-74	.621	37.1	41.0	38.4	1.87	40.2	35.9	39.1	1.63
Kenya	69-79	.839	48.4	48.4	48.4	3.46	45.3	37.2	43.3	2.42
Malawi	66-77	.679	43.9	44.7	44.2	2.70	44.7	37.9	42.9	2.21
Tanzania	67-78	.781	43.9	46.2	44.7	3.18	45.5	39.0	43.8	2.56
Lesotho	66-76	.839	38.1	39.9	38.5	2.26	39.8	35.8	38.8	2.06
Swaziland	66-76	.792	45.4	45.9	45.6	2.91	41.8	35.1	40.0	2.01

Notes: Figures in columns 4 to 11 are percentages.

TABLE 2—ESTIMATES OF BIRTH RATES BY VARIOUS METHODS OF ESTIMATION FOR A NUMBER OF ANGLAPHONE AFRICAN COUNTRIES WITH TWO CENSUSES

Country	*Adjustment by Coale Method*				*Adjustment by Generalized Stable Equations*						*Preston Integrated Method*
	b_s	r_0	r_s	b_e	b_s^*	r_w	r_s^*	b_n	b_n''	b_n'	
(1)	(2)	(3) a	(4) a	(5)	(6)	(7) a	(8) a	(9)	(10)	(11)	(12) b
Ghana	50.8	2.88	3.34	49.1	49.0	2.52	3.18	46.7	50.8	47.8	49.8
Liberia	41.8	3.07	2.34	44.2	37.4	3.08	1.88	40.9	41.5	44.2	49.3
Sierra Leone	47.7	1.87	1.55	48.8	42.4	2.11	1.07	45.8	46.4	49.7	53.9
Kenya	49.0	3.46	3.55	48.7	49.1	3.17	3.56	47.6	48.1	47.6	50.6
Malawi	50.6	2.70	2.35	52.0	50.0	3.17	2.30	53.4	52.8	53.8	54.7
Tanzania	48.0	3.18	3.00	48.7	45.4	3.45	2.73	47.9	48.2	49.7	51.5
Lesotho	37.1	2.26	2.31	36.9	34.0	2.47	1.98	35.3	35.0	37.5	38.2
Swaziland	47.2	2.91	3.01	46.9	46.3	2.99	2.91	46.6	46.5	47.2	49.4

Notes : *a*—Figures are percentages.
b—Figures refer to mid-point of the two censuses and not (T—7.5).

ment. This is because most of the countries studied are close to stability during the intercensal period. But in situations, where mortality and fertility declined during intercensal period and/or migration took place between the ceniuses, the b'_m, estimates will give poor results.

The estimates obtained by the application of Preston integrated method to the data of females in the latest census gave estimates which are on the high side compared to either *be* or b_n. It is important to note that the estimated birth rate by Preston integrated method refers to the time which is mid-point of the two census dates which approximates to time *(T*— 5), when the censuses are ten years apart, whereas the other estimates refer to the time (T—7.5). Thus part of the difference could be accounted for by the difference in the time reference between this method and the others.

Some interesting points emerge from the above. Although, the original method of Coale is based on the simple principle of reverse surviving population under age 15, C(15), using life table corresponding to estimated *15* the method leads to birth rate estimates very close to those obtainable by more complex methods based on generalized stable population equations. This close agreement adds greater satisfaction io the African data situation in the use of the birth rates derived by Coale's method which is very simple to apply and at the same time very general.

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